Team Test, Division B

Gunn Math Competition 2026

Instructions and Format

- This round contains 15 short-answer questions to be solved in 60 minutes as a team. This means that you are not allowed to discuss these problem with anyone outside of your team until Lunch.
- Each problem's point value is shown.
- All answers are integers between 0 and 999, and so responses must be integers in that range.
- You will be given reminders about the time you have remaining. At the end, stop immediately after you are told to.
- Only answers written inside the boxes on the answer sheet will be considered for grading. You will **not** be given additional time to write answers on the answer sheet after the 60 minutes is up.
- NO CALCULATORS (or abaci). Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Thank you to our sponsors. Without them, we would never have been able to make this event possible.

1. What is the remainder when

$$1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 \cdots + 2025 - 2026$$

is divided by 1000? [**35**]

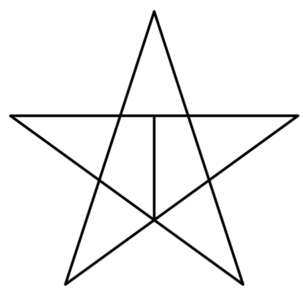
2. The volume of a certain enneacontakaienneagonal prism is 45. What would be the volume of a solid that is identical to the given prism but each dimension is twice as great? [**35**]

3. Let a, b, and c satisfy

$$a + b = 28,$$
$$a + c = 33,$$
$$b + c = 37.$$

Compute $a^2 + b^2 + c^2$. [**40**]

4. How many ways can Grace color the following star using red, yellow, and blue, such that no two same colors are touching the same edge? Rotations and reflections are considered distinct colorings. [**40**]

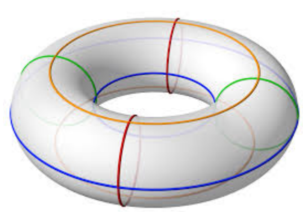


5. Find the smallest positive integer ending in the digit 6 such that when this 6 is moved to the front of the number, the new number is exactly 4 times the original number. [**45**]

6. Find the area of the region defined by all points in the xy plane satisfying, $x, y \geq 0$ and $0 \leq \lfloor x \rfloor + \lfloor y \rfloor \leq 10$. [**45**]

7. Find the greatest common divisor of all numbers of the form $p^4 - 1$, where p is a prime number strictly greater than 5. [**50**]

8. A torus is a donut-shaped object. It is created when a circle is rotated 360 degrees about an axis outside of it, forming a 3D shape. Suppose we have a torus of outer radius 69 and inner radius 60. What is the radius of the smallest possible sphere that we can place on top of the torus such that no part of it is in the bottom half of the torus? [**50**]



9. Let O be the origin, $(0, 0)$ in the xy plane. At time $t = 0$, an ant is at the point $S = (1, 0)$. During the nth minute, the ant's position is rotated counterclockwise by an angle of n° about the origin. Then, (still during the same minute), the ant is rotated n° clockwise about the midpoint of its origin and the position. After 67 minutes, it ends at the position E. What is the value of the (acute) angle $\angle SOE$? **[50]**

10. Neil has a ruler of length 1. For every integer $n \geq 1$, after n minutes have passed, Neil rolls a fair $n-$sided die (a die with n faces numbered 1 through n) and if it lands on a number k, he divides the ruler into k pieces of equal length, keeps one of the pieces, and discards all the others. After 4 minutes, the expected length of the ruler that Neil is holding is $\frac{a}{b}$, where a and b are relatively prime Compute the remainder when $a + b$ is divided by 1000. **[55]**

11. Let k be an integer between 0 and 999. There are exactly 30 integer (possibly negative) values of n that satisfy $n^3 + k^2$ is divisible by $n + k$. Compute k. **[55]**

12. Say we have a set $S = (1, 2, 3, 4, ..., 10)$. An adjacency is defined as when two consecutive numbers are in a set. For example, in the set $(3, 4, 5)$, there are 2 adjacencies, $(3, 4)$ and $(4, 5)$. How many subsets of S have exactly 3 adjacencies? **[60]**

13. A polynomial $P(x)$ of degree 2026 satisfies $P(n) = \frac{1}{n}$ for all integers n from 1 to 2027 inclusive. Find the remainder when $\frac{1}{P(2028)}$ is divided by 1000. **[60]**

14. Find the sum of all positive integers n such that 3^n is a divisor of $(3^{10} + 1)^n - 1$. **[65]**

15. Three circles of radius 1 are externally tangent to each other. A fourth circle is inscribed between them and three more circles of equal radius, r, are inscribed in it (shaded in grey on the diagram). Given that r can be expressed in the form $a - b\sqrt{c}$, where a, b, c are positive integers and c is square-free, compute $a^2 + b^2 + c^2$. **[65]**