# Team Test, Division A

## Gunn Math Competition 2026

## Instructions and Format

- This round contains 10 short-answer questions to be solved in 60 minutes as a team. This means that you are not allowed to discuss these problems with anyone outside of your team until Lunch.
- Each problem's point value is shown.
- All answers are integers between 0 and 999, and so responses must be integers in that range.
- You will be given reminders about the time you have remaining. At the end, stop immediately after you are told to.
- Only answers written inside the boxes on the answer sheet will be considered for grading. You will **not** be given additional time to write answers on the answer sheet after the 60 minutes is up.
- NO CALCULATORS (or abaci). Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Thank you to our sponsors. Without them, we would never have been able to make this event possible.

1. The volume of a certain enneacontakaienneagonal prism is 45. What would be the volume of a solid that is similar to the given prism with each dimension twice as great? **[30]**

2. Find the sum of all positive integers n such that
$$\frac{n^2+20n+64}{n^2+6n+8}$$
is an integer. **[35]**

3. Suppose that $x$ is the positive real number that satisfies
$$x+\frac{1}{x}=3.$$
Compute the remainder when
$$x^{2^{67}}+x^{-2^{67}}$$
is divided by 1000. **[40]**

4. Let $k$ be some integer between 0 and 999. Suppose that there exist exactly 30 different (possibly negative) integers, $n$, such that $n^3+k^2$ is divisible by $n+k$. Find $k$. **[45]**

5. Consider a sphere of radius 2 centered at the origin. Neil starts at the origin, and at any given moment, he can move either in the $x, y$, or $z$ directions (like a rook in 3D). Every minute, he randomly makes a move to another lattice point in or on the sphere, with each possible point being equally likely. What is the expected number of seconds until he is on the sphere? **[50]**

6. Let $\omega_1$ and $\omega_2$ be concentric circles with radii 6 and 7, respectively, and common center $O$. Let $A$ and $C$ be two points on $\omega_2$ such that $AC$ is tangent to $\omega_1$ at $B$. Let $E$ be the foot of the altitude from $C$ to $OA$, and let $D$ be the intersection of lines $OB$ and $CE$. Let $\omega_3$ denote the circumcircle of $\triangle OAD$. $\omega_3$ intersects $\omega_1$ and $\omega_2$ again at points $F$ and $G$, respectively, where $F$ and $G$ are on the opposite side of $OA$ to $C$. Let $\omega_1$ intersects $\omega_3$ again at $H$. The distance between the lines $FH$ and $GA$ is of the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime integers. Compute $m+n$. **[50]**

7. Let $\alpha, \beta$, and $\gamma$ be the roots of
$$x^3+x^2-6x-7=0.$$
Compute
$$-\left(\alpha\left(\beta+\gamma\right)\right)^3-\left(\beta\left(\gamma+\alpha\right)\right)^3-\left(\gamma\left(\alpha+\beta\right)\right)^3. \text{ [55]}$$

8. Five points are chosen uniformly at random on the circumference of a unit circle. Let $p$ be the probability that for some three points $A, B, C$ among the five, one of the angles in $\triangle ABC$ is greater than or equal to $140°$. Then $p$ can be expressed $\frac{m}{n}$, where $m, n$ are relatively prime positive integers. Compute the remainder when $m+n$ is divided by 1000. **[60]**

9. Let $S$ be the set of all ordered 2026-tuples $T$ of nonnegative integers $\{a_1, a_2, a_3, \ldots a_{2025}, a_{2026}\}$ where
$$\sum_{i=1}^{2026} a_i = 2026.$$
Compute the remainder when
$$\sum_{T\in S}\prod_{i=1}^{2026}\binom{2a_i}{a_i}$$
is divided by 1000. **[65]**

10. Let $ABC$ be a triangle such that the side lengths of $AB$ and $AC$ are both integers that differ by more than 2025. Let $\Gamma$ be the circumcircle of $ABC$. Then there exists a distinct circle $\omega$ tangent to sides $AB$ and $AC$, and internally tangent to $\Gamma$ at a point $P$. Given that the area of $PAB$ is exactly two times the area of $PAC$, find the remainder when the minimum possible value of $AB+BC$ is divided by 1000. **[70]**