# Individual Test, Division A

### Gunn Math Competition 2026

## Instructions and Format

- This round contains 10 short-answer questions to be solved in 60 minutes, by yourself. This means that you are not allowed to discuss these problem with anyone outside of your team until Lunch.
- Each problems is worth more points than the previous problems, but not by a significant amount, so be mindful of where you spend your time.
- All answers are integers between 0 and 999, and so answers must be integers in that range.
- You will be given reminders about the time you have remaining. At the end, stop immediately after you are told to.
- Only answers written inside the boxes on the answer sheet will be considered for grading. You will **not** be given additional time to write responses on the answer sheet after the 60 minutes is up.
- NO CALCULATORS (or abaci). Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Thank you to our sponsors. Without them, we would never have been able to make this event possible.

1. Let the operation $x \oplus y$ be defined for real numbers $x$ and $y$, where $xy \neq -1$, as

$$x \oplus y = \frac{x+y}{1+xy}.$$

Then the value of

$$\left(\left(\left(\left(\frac{1}{3} \oplus \frac{1}{2}\right) \oplus \frac{3}{5}\right) \oplus \frac{2}{3}\right) \oplus \frac{5}{7}\right)$$

can be expressed as $\frac{a}{b}$, where $a$ and $b$ are relatively prime positive integers. Compute the remainder when $a+b$ is divided by 1000.

**Answer:** $\boxed{440}$ *(Proposed by: Chanew Kim)*

2. In rectangle $ABCD$, diagonal $AC$ has length 26. A point $P$ lies on segment $AC$ such that $BP$ is perpendicular to $AC$. If $AB = 10$, the length $BP$ is in the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m+n$.

**Answer:** $\boxed{133}$ *(Proposed by: Aarush Rachakonda)*

3. Define a sequence where $a_1 = 1$, and

$$a_{n+1} = \frac{a_n}{1 + n \cdot a_n}$$

for $n \geq 1$. Compute $\frac{1}{a_{26}}$.

**Answer:** $\boxed{326}$ *(Proposed by: Neil Dixit)*

4. Suppose we have two circles of radii $n$ and $n+1$ for integer $n$ that are externally tangent at point $P$. There is another common tangent line to the circles at point $Q$ and $S$ respectively. Compute the minimum $n$ for which the area of the circumcircle of $\triangle PQS$ is greater than 10000.

**Answer:** $\boxed{56}$ *(Proposed by: Neil Dixit)*

5. Let $ABCD$ be a rectangle with $AB = 20$ and $BC = 10$. Let $l_1$ be the line that bisects $\angle ACD$ and $l_2$ be the line that bisects $\angle ADB$ and let them meet at point $P$. For $B, C$, and $D$, define points $Q, R$, and $S$ analogously. The area of quadrilateral $PQRS$ can be expressed in the form $\frac{a-b\sqrt{c}}{d}$, where $a, b, c$, and $d$ are positive integers such that $b$ and $d$ are relatively prime and $c$ is square-free. Find the remainder when $a+b+c+d$ is divided by 1000.

**Answer:** $\boxed{8}$ *(Proposed by: Alex Tsagaan)*

6. On Arastradero Road, there are 3 distinct houses lined up in a row. In the surrounding area, there are 4 distinct utility stations. Each of the 3 houses picks a non-empty subset of the utility stations, and connects a (not necessarily straight) wire to each of those stations in the subset. These wires can be infinitely long. Let $P$ be the number of ways the 3 houses can choose stations such that the wires can be placed with none of them crossing with each other, except at the endpoints. Find the remainder when $P$ is divided by 1000.

**Answer:** $\boxed{346}$ *(Proposed by: Alex Tsagaan)*

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7. Consider the set, $S$, of integers $\{1, 2, 3, \ldots, 11!\}$. Let $X$ denote a randomly selected number in $S$. Compute the expected value of $\gcd\{X, 11!\}$.

   **Answer:** $\boxed{169}$ *(Proposed by: Neil Dixit)*

8. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $n$ denote the number of pairs of subsets $(A, B)$ of $S$ such that $A \subseteq B$ and the sum of elements that are in $B$ but not in $A$ is divisible by 11. Compute the remainder when $n$ is divided by 1000.

   **Answer:** $\boxed{989}$ *(Proposed by: Neil Dixit)*

9. (Problem 9 was voided.)

10. There exists a unique triple of integers $(a, b, c)$ such that $0 < a < b < c < 90$, no two of $a, b, c$ sum to 90, and

$$\tan(2026^\circ) = \tan(a^\circ) \cdot \tan(b^\circ) \cdot \tan(c^\circ).$$

   Compute the remainder when $abc$ is divided by 1000.

   **Answer:** $\boxed{72}$ *(Proposed by: Alex Tsagaan)*