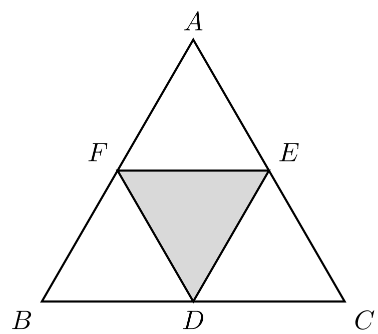
Set 1 [10]

1. Yu has 100 apples, and You have none. You take away 33 apples from Yu. How many apples do You now have?

2. An apple weighs 200 grams. John can lift 14500 grams. If his basket weighs 600 grams, how many apples can he hold assuming he puts them in his basket?

3. Let $\triangle ABC$ be an equilateral triangle with area 268. Points D, E, F are the midpoints of sides BC, CA, AB, respectively. What is the area of $\triangle DEF$?



Set 2 [11]

4. Grace is broke and only has \$2.35. Being a generous person, she is willing to give ALL her money to the person who guesses how many nickels (5 cents) and quarters (25 cents) they have. Grace notices you struggling and gives three extra pieces of information:

   (a) The total number of coins is prime.

   (b) The number of quarters can be written as the product of 2 primes.

   (c) The total number of coins can be written as the sum of 10 times one of the primes in (b) plus the other prime in (b).

   What is the number of nickels?

5. Given that $x + \frac{1}{x} = 29$, compute $x^2 + \frac{1}{x^2}$.

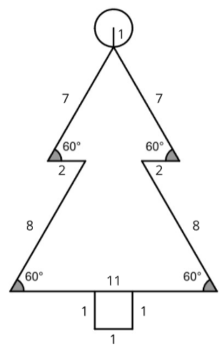
6. A Pythagorean triplet consists of 3 integers that form the sides of a right-angled triangle. Let n be the largest possible integer that is in a Pythagorean triplet with 67. Compute the remainder when n is divided by 1000.

Set 3 [13]

7. Compute the unique 3-digit integer, n, such that $n(n-1)(n-2)$ is divisible by 60000.

8. There are 196 countries in the world, and 14 in Oceania. There are 6 continents with countries: Europe, Asia, North America, South America, Africa, and Oceania. For a school project, Alex B and Alex T were asked to choose a country, and both decided to choose one randomly. Alex B decides to to randomly choose the country out of the 196. In contrast, Alex T decides to first randomly choose a continent out of the 6, then randomly chooses a country in that chosen continent. Let P denote the difference between the probability of Alex B choosing Australia and of Alex T choosing Australia, given that Australia is in Oceania. Compute $\frac{1}{P}$.

9. Olivia finds a 2D tree drawn on an old Christmas card. A circle tops it off, and a square base supports it. All lengths and angles are as labeled, and the tree has a vertical line of symmetry. The area of the tree, in square centimeters, can be expressed as $a\pi + b + \frac{c\sqrt{d}}{e}$,where a, b, c, d, and e are positive integers such that d is square-free and c is relatively prime to e. Compute $a + b + c + d + e$.



Set 4 [14]

10. Mary has 10 pairs of gloves. Oh no! They got mixed up in her drawer! Two pairs are red, three pairs are green, three pairs are blue and two pairs are purple. Mary also counts a purple and a blue glove as a pair, since they look similar enough from far away. The chances Mary has a pair of gloves after randomly taking three gloves out (without replacement) can then be expressed as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.

11. The positive number, x, that satisfies

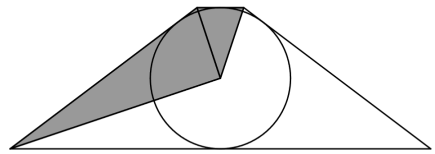
$$\sqrt{x-1} + \sqrt{x+10} = 5$$

can be represented in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Compute $a + b$.

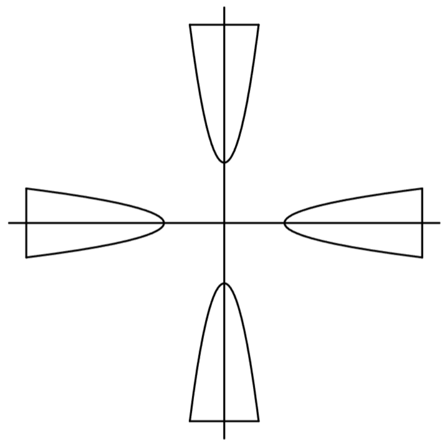
12. Consider a set of points with each point labelled with a (distinct) factor of 360. You can travel from one point, a, to another point, b, if and only if b is a divisor of a and $b \neq a$. The cost to make this journey is b. Find the cost of the most expensive path possible from 360 to 1. For example, one possible path would be $360 \to 120 \to 24 \to 6 \to 1$ and summing up the cost of each journey gives $120 + 24 + 6 + 1 = 151$.

Set 5 [16]

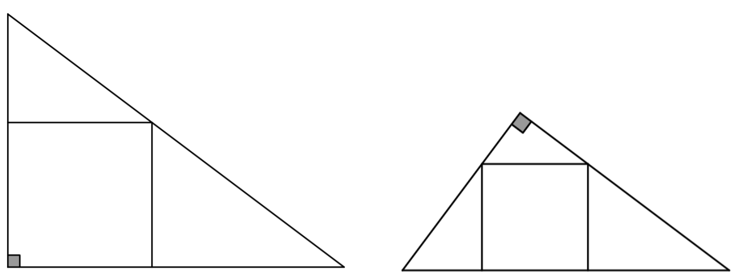
13. Let $ABCD$ be an isosceles trapezoid with $AB \parallel CD, BC = AD$, and $AB = 8$. A circle, ω, is inscribed within $ABCD$ (internally tangent to all 4 sides), and has center I. Given that the area of $\triangle BCI$ is 5 times the area of $\triangle ABI$, compute the area of the trapezoid.



14. Let R denote the region above the parabola $y = x^2 + 7$ and below the line $y = 23$. Let S denote the region formed when you rotate R a full revolution ($360°$) about the origin in the xy plane. The diagram shows the region at a few positions while it is rotated. The area of S is $n\pi$, where n is an integer. Compute n.



15. Consider a right-angled triangle with side lengths $3, 4$, and 5. You inscribe a square in it in 2 different ways shown in the diagram. The absolute value of the difference between the two squares' side lengths is $\frac{a}{b}$ where a and b are relatively prime positive integers. Compute $a + b$.



Set 6 [17]

16. Find the sum (in base 10) of all unique prime factors of the following number (which is in base 6):

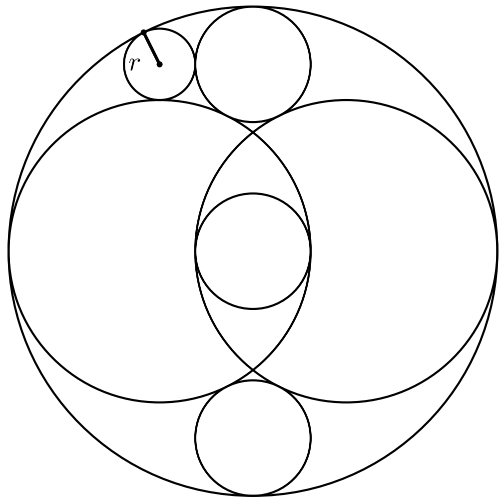
$$1111111111_6.$$

17. A rectangular prism is such that the sum of the lengths of its edges is 80 and its longest diagonal has length 12. A new rectangular prism is formed by increasing each dimension of the old rectangular prism by 1. Compute the surface area of the new rectangular prism.

18. Take a sector of a circle of radius 14, and fold it so that it makes a cone with an open base (like an ice-cream cone). To that open base, attach a hemisphere. Given that this new shape's surface area is 156π, its volume can be expressed in the form $\pi(a\sqrt{b} + c)$, where a, b, and c are positive integers and c is square-free. Compute $a + b + c$.

Set 7 [19]

19. Two circles, A and B, of radii 1 intersect twice, and a third circle, C, of radius $\frac{3-\sqrt{5}}{2}$ is inscribed in their intersection. Two other circles, D and E, also of radius $\frac{3-\sqrt{5}}{2}$ are above and below the region of intersection, and are tangent to A and B. There is a circle F, that is tangent to A, B, D, and E, and it has radius $\frac{1+\sqrt{5}}{2}$. There's a circle tangent to A, D, and F with radius r. Compute $\lfloor 100r \rfloor$.



20. The digits of 3! do not end in a 0. The digits of 6! end in one zero. The digits of 12! end in two zeroes. For how many integers from 0 to 999, is there a factorial that ends in exactly that many zeroes?

21. The positive value of x such that

$$x^2 + \frac{x^2}{(x+1)^2} = 63$$

is equal to $\frac{a+\sqrt{b}}{c}$ where a, b, c are positive integers such that b is square-free and a and c are relatively prime. Compute $a+b+c$.

Set 8 [20]

22. Neil starts at the origin in the xy plane. If Neil is at (x, y), he can move to $(x+1, y)$ or $(x, y+1)$. Neil wants to only go through points where the x and y coordinate are equal or relatively prime. Let A be the number of paths Neil can take to reach $(67, 67)$. Estimate A.

23. Consider the set of all possible triangles with its greatest side-length less than 2025. If I choose one of these triangles at random, the expected value of its area is B. Estimate B.

24. Let C be the number of triangles with 3 integer side lengths all less than or equal to 2026 where at least one angle in the triangle, when rounded to the nearest degree, is $67°$. Estimate C.