

Tiebreaker, Division B

Division B, 15 Minutes, Individual

1. Define $\text{ord}_n(x)$ as the minimum integer k such that $x^k \equiv 1 \pmod{n}$. Compute

$$\sum_{x=1}^6 \text{ord}_7(x).$$

Answer: $\boxed{21}$ (*Proposed by: Neil Dixit*)

2. A sphere of radius 1 with center O is inscribed in a cone. Let P be a point of tangency not on the base. Suppose that the line OP intersects the base of the cone on its circumference. Let V be the volume of the cone. Compute

$$[100V].$$

Answer: $\boxed{942}$ (*Proposed by: Neil Dixit*)

3. Compute

$$\left\lfloor 100 \sum_{n=0}^{\infty} \arctan \left(\frac{1}{n^2 + n + 1} \right) \right\rfloor.$$

Answer: $\boxed{157}$ (*Proposed by: Neil Dixit*)