# Team Test (Division B)

## Gunn Math Competition 2025

## Instructions and Basic Format

- This round contains 15 short-answer questions to be solved in 60 minutes, by you and your team. This means you are not allowed to discuss this problem with anyone outside of your team until Lunch. Each problem is worth more points than all previous points, but by no significant amount, so be mindful of where you spend your time. All answers must be expressed in simplest form unless specified otherwise. Only Answers written inside the boxes on the answer sheet will be considered for grading.
- **NO CALCULATORS (or abaci).** Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as $7^8$.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as $\pi$, $e$, $\sin 10^\circ$, and $\ln 2$ in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

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# Problems

1. Let $f(\text{`abc'}) = \text{`def'}$, where $d$ is the remainder when $a+b$ is divided by 10, $e = b$, and $f$ is the remainder when $b+c$ is divided by 10. Both the input and output from the function must have 3 digits, with no leading zeroes. For how many integers is $f(f(\text{`abc'})) = \text{`abc'}$?

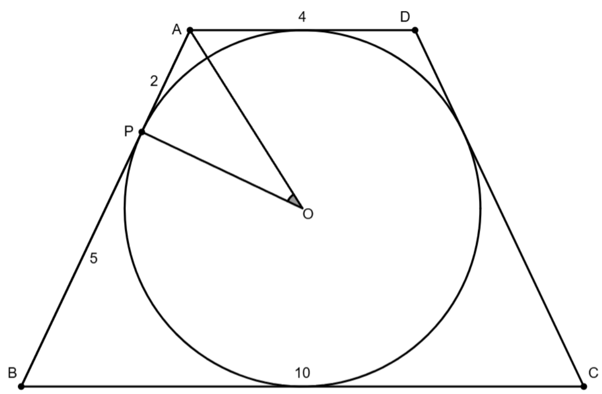
   Note: The letters represent digits from 0 to 9, not variables. For example 'abc' $= 100a + 10b + c$ and 'def' $= 100d + 10e + f$.

2. Given that the western Roman empire fell to Germanic tribes in 476 AD, compute

$$\frac{1^2 - 2^2 + 3^2 - 4^2 .... + 2025^2}{2025}.$$

   The historical information has nothing to do with the problem, but Jerry found it funny so it is a part of the problem anyway.

3. $ABCD$ is an isosceles trapezoid with $AD \parallel BC$. $\overline{AD} = 4, \overline{BC} = 10$, and a circle $\omega$ with center $O$ is inscribed inside $ABCD$ (see diagram). $\omega$ is tangent to $AB$ at point $P$, where $\overline{AP} = 2$ and $\overline{BP} = 5$. Compute $\cos^2(\angle AOP)$.




4. Anna and Beatrice are playing tennis, where Anna has a 75% chance of winning any given point. The first person to reach 4 points wins the match (if it reaches 3-3, then the winner of the next point wins the match). Compute the square root of the probability that Beatrice wins.

5. Let $\alpha, \beta$, and $\gamma$ be the roots of $f(x)$. Given the following system of equations, compute $f(1)$:

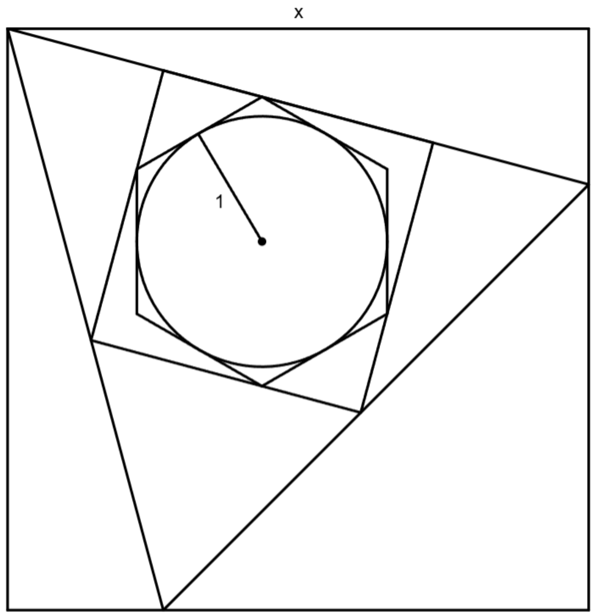
$$\alpha + \beta + \gamma = 7$$
$$\alpha^2 + \beta^2 + \gamma^2 = 29$$
$$\alpha^3 + \beta^3 + \gamma^3 = 103.$$

6. Let $a$ denote the number of ways to arrange the letters in "GUNNMATHCOMP" so that **exactly** one of the substrings "GUNN", "MATH", or "COMP" appear in the arrangement. Let $b$ denote the number of ways so that **at least** two of the substrings appears in the arrangement. Calculate the value of $\frac{a}{b}$ to the nearest integer.

7. An ant starts at the origin, $O$, initially facing East. The ant moves 1 unit East, then turns 60 degrees counterclockwise and moves $\frac{1}{3}$ of a unit. The ant then turns 60 degrees counterclockwise again and moves $\frac{1}{9}$ of a unit, and so on and so forth, turning an infinite amount of times. It eventually ends up at the point $P$. Compute $\overline{OP}^2$.

8. Suppose you have a tetrahedron with 3 sides of side length 2, and 3 sides of side length 3 with an equilateral triangle of side length 2 in the base. The radius of the largest possible sphere inscribed in the tetrahedron can be expressed in the form $\frac{a\sqrt{b}-\sqrt{c}}{d}$, where $b$ and $c$ are square-free positive integers and $a$ and $d$ are positive integers. Compute $a+b+c+d$.

9. A circle of radius 1 is inscribed in a regular hexagon which is inscribed in the smallest possible square which is inscribed in the smallest possible equilateral triangle which is inscribed in the smallest possible square. Compute the side length, $x$, of the outer square.




10. Compute the following, all angles are in degrees:

$$\sum_{i=1}^{89}\left(\frac{1}{1+\cos(i)}+\frac{1}{1+\sin(i)}+\frac{1}{1+\tan(i)}+\frac{1}{1+\sec(i)}+\frac{1}{1+\csc(i)}+\frac{1}{1+\cot(i)}\right).$$

11. Given 2 circles, $\omega_1$ and $\omega_2$, each of radius 1, and with centers $O_1$ and $O_2$. They intersect twice and have a common tangent. Consider a 3rd circle, $\omega_3$ that is tangent to this common tangent and $\omega_1$ and $\omega_2$. Let $x$ be the length of $O_1O_2$ that is **not** in the intersection of $\omega_1$ and $\omega_2$. Consider $l$, the line tangent to both $\omega_1$ and $\omega_3$ at a common point. Given that $l$ intersects $O_1O_2$ a distance of $\frac{1}{8}$ away from $O_2$, compute $x^2$.

12. Suppose we have a $3\times n$ grid and we want to tile it with $3\times 1$ and $1\times 3$ dominoes. What is the minimum $n$ such that we can tile the grid in at least 2025 ways?

13. Find the minimum positive integer, $n$, such that $7^n - 2^n$ is divisible by 2025.

14. Given that $a, b$, and $c$ are positive, real numbers such that $a+b+c=21$, consider the minimum possible value of

$$\frac{1}{a+1}+\frac{7}{b+1}+\frac{49}{c+1}.$$

It can be expressed in the form $\frac{a+b\sqrt{c}}{d}$, where $a, b, c$, and $d$ are positive integers where $c$ is square-free and $a$ and $d$ are relatively prime. Compute $a+b+c+d$.

15. Let $\omega_1$, a circle centered at $O_1$ with radius 6, and $\omega_2$, a circle centered at $O_2$ with radius 8, intersect at $A$ and $B$. Define $C$ as the second intersection between $\omega_1$ and the line tangent to $\omega_2$ at $A$. Define

$D$ as the second intersection between $\omega_2$ and the line tangent to $\omega_1$ at $A$. Let line $CD$ intersect the circumcircle of $AO_1O_2$ at $E$ and $F$, where $E$ is closer to $C$. Given that $O_1A = O_2F$ and $O_1O_2$ is parallel to $BF$, compute the area of quadrilateral $ABFO_2$.