Team Test (Division A)

Gunn Math Competition 2025

Instructions and Basic Format

- This round contains 10 short-answer questions to be solved in 60 minutes, by you and your team. This means you are not allowed to discuss this problem with anyone outside of your team until Lunch. Each problem is worth more points than all previous points, but by no significant amount, so be mindful of where you spend your time. All answers must be expressed in simplest form unless specified otherwise. Only Answers written inside the boxes on the answer sheet will be considered for grading.
- **NO CALCULATORS (or abaci).** Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2}+\frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π, e, $\sin 10^\circ$, and $\ln 2$ in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

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Problems

1. $ABCD$ is an isosceles trapezoid with $AB \parallel CD$. $\overline{AB} = 4, \overline{CD} = 10$, and a circle ω with center O is inscribed inside $ABCD$. ω is tangent to AD at point P, where $\overline{AP} = 2$ and $\overline{DP} = 5$. Calculate $\cos^2(\angle AOP)$.

2. In the middle of the Earth Tectonics labs, the power goes off in Chanew's chemistry classroom. Dr. Mellows (his teacher) has three red lava lamps and three blue lava lamps, all of which are battery-powered. She arranges them in a row on her desk randomly, and then randomly turns three of them on. What is the probability that the leftmost lamp is blue and off, and the rightmost lamp is red and on?

3. Let a denote the number of ways to arrange the letters in "GUNNMATHCOMP" so that **exactly** one of the substrings "GUNN", "MATH", or "COMP" appear in the arrangement. Let b denote the number of ways so that **at least** two of the substrings appear in the arrangement. Calculate the value of $\frac{a}{b}$ to the nearest integer.

4. Suppose you have a polynomial, $p(x)$, of degree 4 with integer coefficients such that $p(0) = 7, p(1) = 11$, and $p(2) = 13$. Compute the minimum possible positive value of $p(25)$.

5. Let points A, B, and C lie on the curve $y = x^2$, and $B = (4, 16)$. The x-coordinates of A and C sum to 8, and $\angle ABC = 90^o$. Let $G = (G_x, G_y)$ be the centroid of $\triangle ABC$ and O be the origin. Given that $G_x + G_y$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, what is $m + n$?

6. Suppose you have a circle of radius 1 centered at the origin. Given two points a known distance x away from the origin, randomly chosen, the probability that the midpoint of the 2 points lies inside or on the circle is $\frac{1}{6}$. Find x.

7. Suppose you have a tetrahedron with 3 sides of side length 2, and 3 sides of side length 3 where the base is an equilateral triangle of side length 2. The radius of the largest possible sphere that can be inscribed in the tetrahedron can be expressed in the form $\frac{a\sqrt{b}-\sqrt{c}}{d}$, where b and c are square-free positive integers, and a and d are positive integers. Find $a + b + c + d$.

8. Alice and Bob are worst enemies in a class of six students. Each student has at least one friend, no two students share more than one friend, and no three students are all friends with each other. In how many ways can the six students be friends with each other?

9. Let ω_1, a circle centered at O_1 with radius 6, and ω_2, a circle centered at O_2 with radius 8, intersect at A and B. Define C as the second intersection between ω_1 and the line tangent to ω_2 at A. Define D as the second intersection between ω_2 and the line tangent to ω_1 at A. Let line CD intersect the circumcircle of AO_1O_2 at E and F, where E is closer to C. Given that $O_1A = O_2F$ and $\overline{O_1O_2} \| \overline{BF}$, compute the area of quadrilateral $ABFO_2$.

10. Let $f(n)$ denote the number of solutions $(a_1, a_2, \ldots, a_{4n})$ to the congruence

$$\sum_{i=1}^{4n} a_i^2 \equiv 1 \pmod{2025}.$$

where all a_i are between 1 and 2025. Find the unique positive integer n such that

$$\sum_{k=2,3,5} \nu_k\left(f(n)\right) = 2021.$$