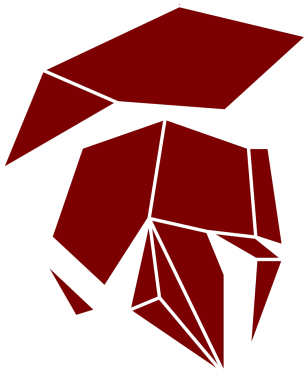
Individual Test (Division B)

Gunn Math Competition 2025



Instructions and Basic Format

- This round contains 10 short-answer questions to be solved in 60 minutes, by yourself. This means you are not allowed to discuss this problem with anyone outside of your team until Lunch. Each problem is worth more points than all previous points, but by no significant amount, so be mindful of where you spend your time. All answers must be expressed in simplest form unless specified otherwise. Only Answers written inside the boxes on the answer sheet will be considered for grading.
- **NO CALCULATORS (or abaci).** Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2} + \frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π, e, $\sin 10^\circ$, and $\ln 2$ in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

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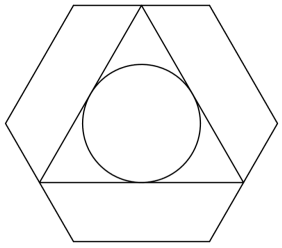
Problems

1. Express the recurring decimal $0.\overline{01}$ as a fraction.

2. There are 500 freshmen at Gunn High School, who can take Spanish, French, or Chinese. Students may took none, one, two, or all three of the languages. 300 are taking Spanish, 250 are taking French, and 100 are taking Chinese. 100 are taking French and Spanish, 60 are taking Spanish and Chinese, and 30 are taking French and Chinese. 10 students are taking all three languages. How many students are not studying a foreign language?

   Note that a student who is taking all three languages is counted as taking French and Spanish, Spanish and Chinese, French and Chinese, and French, and Spanish, and Chinese. A student who is counted as taking French may also be taking Spanish and/or Chinese.

3. A circle is inscribed in an equilateral triangle, which is inscribed in a regular hexagon, and all three shapes are concentric, as depicted in the diagram. Compute the area of the circle divided by the area of the entire hexagon (which includes the area of the triangle and circle inside it).

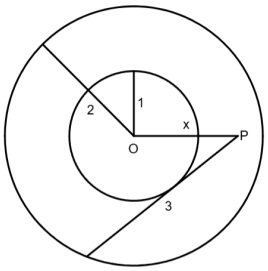


4. Satvik wants to reach the cafeteria. He starts at the origin and if he is at position (x, y), he can only move to positions $(x+1, y+1)$,$(x+2, y)$, or $(x, y+2)$. For example, if he is at the point $(3, 5)$ at some instant, he can move to $(5, 5)$, $(3, 7)$, or $(4, 6)$. How many paths can Satvik take to reach the position $(8, 8)$ so he can eat lunch?

5. For how many positive integer values of x less than 1000 is the following expression rational (can be expressed as a fraction with integers in both the numerator and denominator)?

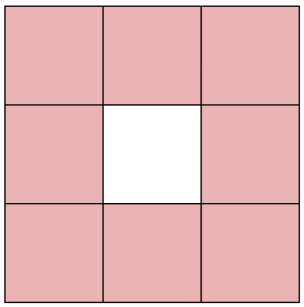
$$\sqrt{x^5 + x^4 + x^3 + x^2 + x + 1 + \frac{1}{x-1}}.$$

6. Consider two circles of radii 1 and 2 with the same origin, O. Choose a point, P that is inside the larger circle but outside the smaller circle, and is a distance x from their centers. Draw a line segment starting at P that is tangent to the smaller circle, and has the other endpoint on the outer circle. Given that the line segment's length is 3 compute x^2.



7. Samuel writes the numbers 1 to 2025 in binary (base 2) on a piece of paper. How many zeroes does he write down in total (no leading zeroes of course)? For example, when writing the number 23 in base 2, Samuel would write 10111, meaning he writes down 1 zero. When writing the number 1367, he would write 10101010111, meaning he writes down 4 zeroes.

8. Suppose that the region R consists of a 3×3 square without the middle square, and suppose two points are randomly chosen in R. Compute the probability that their midpoints lies outside of R.



9. Consider $\triangle ABC$ where $\overline{AC} = 31$, $\overline{BC} = 30$, and $\overline{AB} = 29$. Let D be a point inside $\triangle ABC$ such that $\angle DAC = \angle ACB$, and $\angle ADB = 90^\circ + \angle ACB$. Extend line AD until it intersects BC at point E. Compute $\frac{AD}{BE}$.

10. Define the sequence a_n recursively as follows:
$a_0 = 0$, $a_1 = 1$, and $a_{n+2} = a_{n+1} + xa_n$ for all $n \geq 2$ and some real x. Let S denote the following sum:

$$\sum_{n=1}^{\infty} \frac{a_n}{10^n}.$$

Compute the largest integer value of x such that S is a finite, positive number.