



# GUNN MATH COMPETITION

## Team Round B

Division A // 60 Minutes // March 24TH, 2024

- (20 points) Let  $a \oplus b = ab + 3$ . If  $(2 \oplus x) \oplus 3 = 72$ , compute  $x$ .
- (25 points) What is the ones digit of  $1 + 3 + 5 + \dots + 99$ ?
- (30 points) The distinct real numbers  $-\sqrt{-x}$ ,  $x$ , and  $-x$  form an arithmetic sequence in that order. What is  $x$ ?
- (30 points) How many positive integers  $n$  satisfy  $n^{\frac{1}{n}} = \sqrt{2}$ ?
- (30 points) In rectangle  $ABCD$ , square  $ABFE$  is drawn with  $E$  lying on  $AD$ . Then, square  $EFHG$  is drawn with  $G$  lying on  $ED$ . Rectangle  $CDGH$  is similar to rectangle  $ABCD$ . Compute  $\frac{BC}{AB}$ .
- (35 points) Find the maximum possible area of a right triangle with hypotenuse 7.
- (35 points) The hands of a clock are currently 20 degrees apart. The minute hand is ahead of the hour hand. How many minutes will pass until the earliest time their hands form 53 degrees?
- (35 points) If  $f(x, y) = 3x^2 + 3xy + 1$  and  $f(a, b) + 1 = f(b, a) = 42$ , then determine  $|a + b|$ .
- (35 points) How many (possibly empty) subsets of  $\{1, 2, \dots, 10\}$  have a sum that is at most 27?
- (40 points) Circles with centers  $P, Q$  and  $R$ , having radii 1, 2 and 3, respectively, lie on the same side of line  $l$  and are tangent to  $l$  at  $P', Q'$  and  $R'$ , respectively, with  $Q'$  between  $P'$  and  $R'$ . The circle with center  $Q$  is externally tangent to each of the other two circles. What is the area of triangle  $PQR$ ?
- (45 points) Let  $S_1$  be the set of all integers that can be expressed in  $2^a 3^b$ , where  $a + b$  is even, and let  $S_2$  be all integers that can be expressed in  $2^a 3^b$ , where  $a + b$  is odd. What is

$$\left( \sum_{s \in S_1} \frac{1}{s} \right) - \left( \sum_{s \in S_2} \frac{1}{s} \right)?$$

- (50 points) There are 10 chairs ordered in a line at the doctor's office. Due to social distancing rules, any pair of people must be separated by at least 1 chair. How many ways can any number of people sit in the chairs while maintaining social distancing?
- (50 points) How many base-10 numbers have all nonzero digits that sum to 12?
- (55 points) In triangle  $ABC$ ,  $D$ ,  $E$ , and  $F$  are points on  $BC$ ,  $AC$ , and  $AB$  such that  $AD$ ,  $BE$ , and  $CF$  intersect at  $X$ . Additionally,  $AX = DX$  and  $BX = 4EX$ . Let  $[ABC]$  denote the area of a triangle  $ABC$ . Compute  $\frac{[DEF]}{[ABC]}$ .
- (60 points) Let there be a homecoming game with four teams (Freshman, Sophomore, Junior, and Senior), and the results are decided through a 4-team single-elimination tournament. Assuming the better team is going to win with a  $\frac{4}{5}$  chance, what is the probability that the second best team is going to win the tournament? (Semifinal matches are randomized.)