GUNN MATH COMPETITION

Individual Round B

Division B // 60 Minutes // March 24TH, 2024

1. A circle passes through each vertex of a rectangle with side lengths 2 and 4. What is its radius?

2. There is a circular track of radius 20 m. A cat and dog begin running simultaneously from the same point, in the same direction, at speeds of 3 m/s and 5 m/s, respectively. How many seconds will it take for the cat and dog to meet again?

3. The product of two numbers is 1 and their sum is 3. Compute the larger of the two numbers.

4. How many 5-digit numbers are there in base-3?

5. Suppose a, b, and c are real numbers such that

$$\left(a + \frac{1}{b}\right)\left(b + \frac{1}{c}\right)\left(c + \frac{1}{a}\right) = \left(1 + \frac{1}{a}\right)\left(1 + \frac{1}{b}\right)\left(1 + \frac{1}{c}\right)$$

If $abc = 11$, what is $a + b + c$?

6. Ezra rolls a regular seven sided dice (numbered 1-7) and Jerry rolls a regular six sided dice (numbered 1-6). What is the probability that the product of the two numbers that they get is a multiple of 3?

7. How many paths are there from (0,0) to (6,6) on a coordinate plane that pass through (2,3) if, at each step, you can only move 1 unit up or 1 unit right?

8. Let $ABCDEF$ be a hexagon with unit side length. Find the ratio of the area of the intersection of $\triangle ACE$ and $\triangle BDF$ to the area of the hexagon $ABCDEF$.

9. Suppose a and b are two integers such that $\frac{ab+1}{a-2b} = 11$. What is the maximum possible value of ab?

10. In right triangle ABC, let M be the midpoint of hypotenuse BC. Suppose the incircle of triangle ABM is tangent to AM at X, while the incircle of triangle ACM is tangent to AM at Y. If $BC^2 = 218$ and $XY = 3$, compute the area of triangle ABC.