Division B Set 1 $\left(4 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Bill is asked to compute 23×57, but computes 27×53 instead. What is the positive difference between his answer and the actual answer?
2. Farmer John has 10 cows, each of which produce 6 quarts of milk per day. If there are 4 quarts in a gallon, how many gallons of milk do his 10 cows produce in 8 days?
3. What is the probability that, when two dice are rolled, their sum is even?

Division B Set 2 $\left(5 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Let ABC be a triangle with area 12. Let D be the midpoint of BC, E be the midpoint of AC, and F be the midpoint of AB. What is the area of triangle DEF?
2. Richard has a blue, red, and black sock, two different pants, and three shirts. How many different ways can he dress up, given that he must wear a sock on each of his feet, a pair of pants, and a shirt?
3. Compute $1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + \cdots + 97 + 98 - 99$.

Division B Set 3 $\left(6 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Let point M be on segment BC of $\triangle ABC$ so that $AM = 3$, $BM = 4$, and $CM = 5$. Find the largest possible area of $\triangle ABC$.
2. 7 distinct lines intersect at n distinct points. Find the product of all possible values of n.
3. How many times per day do the minute and hour hand of a clock coincide?

Division B Set 4 $\left(7 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Square $ABCD$ has side length 10. Point E is on BC such that $BE = 6$. Point F is on AD such that the ratio of areas $ABEF$ and $ECDF$ is 2. Find FD.
2. How many ways are there to color a 2×2 grid with 4 colors, such that no two cells that share an edge have the same color? Rotations and reflections are considered distinct.
3. For how many integers n is $\frac{n}{30-n}$ the square of an integer?

Division B Set 5 $\left(8 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Find the number of ways to arrange the numbers $1, 2, \ldots, 8$ such that no two adjacent numbers share a prime factor.
2. A divisor d of a number is *unitary* if it has the property $\gcd\left(d, \frac{n}{d}\right) = 1$. What is the sum of all unitary divisors of 1620?
3. Evaluate the infinite fraction
$$F = \frac{1}{(1-a_1)\frac{1}{(1-a_2)\frac{1}{1-\ldots}}},$$
where $a_i = i$, if $i > 1$ and divides 14 and $a_i = 0$ otherwise.

Division B Set 6 $\left(10 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Call a pair of positive integers (a, b) with $a > 2$ *nice* if for all numbers $\overline{wxyz}_a + \overline{wx}_a \equiv \overline{yz} \pmod{b}$. Find the sum of b across all *nice* pairs (a, b) such that $a < 10$.
2. What is $\sum_{n=0}^{\infty} \frac{5^n + 5^{n-1}4^1 + 5^{n-2}4^2 + \cdots + 4^n}{20^n}$
3. Let C be the sphere $x^2 + y^2 + (z-1)^2 = 1$. Point P on C is $(0, 0, 2)$. Let $Q = (14, 5, 0)$. If PQ intersects C again at Q', then find the length PQ'.

Division B Set 7 $\left(12 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Rectangle $ABCD$ has $AB = 2$ and $BC = 4$. Initially, the rectangle lies flat on the ground. Then, vertex C is held 2 units off the ground while vertex A is fixed in place so that $ABCD$ can rock back and forth with AC as the axis of rotation. The total angle that $ABCD$ can rotate is θ. Compute $\tan\theta$.
2. Brandon the painter wants to paint five consecutive houses on a street. He has red, blue, and yellow paint, but he is not allowed to paint two adjacent houses yellow and blue. In how many ways can he paint the five houses?
3. In a non-square rectangle, construct the diagonals, and for each pair of midpoints of the sides, draw a line between them. This should divide the rectangle into 16 smaller triangles. Using the constructed line segments, how many resulting similar triangle pairs are there?

Division B Set 8 $\left(15 \frac{\textbf{points}}{\textbf{problem}}\right)$

1. Let $f(x) = \lfloor \frac{x}{2.7} \rfloor$. Find the sum of all integers a such that $f(f(f(a))) = 1$.
2. Suppose a faulty coin flips heads $\frac{1}{3}$ of the time and tails $\frac{2}{3}$ of the time. What is the probability that you land heads 3 times before landing tails 3 times?
3. Consider a unit cube in 7 dimensions. A diagonal is defined as any line between two corners that are not connected by an edge. How many diagonals are at least 2 in length?