Tiebreaks B

1. Let $\frac{m}{n}$ be
$$\prod_{n=1}^{\infty}\left(1+\frac{1}{3^{2^n}}\right)$$
If m and n are relatively prime, find $m+n$.

2. $\triangle ABC$ is constructed with $AC = 6$, $BC = 10$, and $\angle C = 30^\circ$. Then $\triangle ABC$ is rotated 345° around C. The area of the shape swept out by the triangle can be expressed in the form $\frac{x\pi+y}{z}$ where x, y, and z are all positive integers and share no common factors greater than 1. Find $x+y+z$.

3. Let $p_2(n)$ be the exponent of the largest power of 2 that divides n. ($p_2(48) = 4$ and $p_2(5) = 0$.) Let m be the number of finite sequences $a_1, a_2, \ldots$ (with possibly length 0) such that $1 \le a_i \le 256$ for all $i \ge 1$ and $p_2(a_j) > p_2(a_{j-1})$ for all $j \ge 2$. Find the sum of distinct prime factors of m.