

Division B Team Problems

- (20 points) If it takes 10 workers 9 days to build the Great Wall of Gunn, how many days will it take 15 workers to build it?
- (20 points) What is the sum of all x such that $x^{x-2} = x^3$?
- (25 points) Eight $2\text{ cm} \times 2\text{ cm} \times 2\text{ cm}$ cubes are removed from the corners of a $5\text{ cm} \times 5\text{ cm} \times 5\text{ cm}$ cube. What is the surface area of the resulting object?
- (25 points) The four points $(12, 12)$, $(3, 5)$, $(10, 14)$, and $(19, x)$ define a rhombus in the Cartesian plane. What is x ?
- (25 points) Suppose $a - 5$, a , and $a + 10$ are all primes. What is the value of $a(a + 10)^2$?
- (35 points) The increasing sequence $1, 5, 6, 25, 26, 30, 31, \dots$ consists of all positive integers that are powers of 5 or sums of distinct powers of 5. What is the 50th number of this sequence?
- (35 points) What is the area of the triangle where the lengths of the sides are equal to the roots of the equation $x^3 - 12x^2 + 47x - 60$?
- (40 points) Consider the function $f(x) = 1 - \frac{1}{x}$. Compute $f^{2023}(2023)$. Express your answer as a common fraction. (Note: $f^n(x) = f(f(\dots f(x)\dots))$, where f is applied n times to x).
- (40 points) Let $ABCD$ be a square inscribed in a circle (its vertices lie on the circle). Let M be the midpoint of arc BC , and let AM and DM intersect BC at points X and Y . Given that the side length of the square is 1, find XY . Express your answer in the simplest radical form.
- (45 points) Aidan the farmer is on a 3-by-3 square field (coordinates $[0, 3]^2$), on a mission to find "Who asked?" However, Angela the ringmaster doesn't want Aidan to discover the truth and decides to randomly place obstacles at three distinct, random, lattice points in the square field to prevent Aidan on his journey. Aidan can only move "up" and "right," and his mission involves getting from the point $(0, 0)$ to $(3, 3)$. What is the probability that Aidan can complete his mission? *Note that if there is an obstacle at $(0, 0)$ or $(3, 3)$, Aidan cannot complete his mission.*
- (50 points) A *unit fraction* is a fraction with a numerator of 1 and an integral denominator. What is the sum of all positive unit fractions that have a finite number of digits after the decimal point?
- (55 points) A $3 \times 5 \times 6$ rectangular prism is completely painted on its outside and then cut into $1 \times 1 \times 1$ cubes. A random cube is then selected and rolled. Given that the cube lands with a painted face up, what is the probability that the chosen cube has three of its sides painted?

13. (55 points) Let 3^n be the largest power of 3 that divides $2023!!$. Find n . Note: $k!! = k(k-2)(k-4)\dots(3)(1)$ for odd k .
14. (60 points) What is the remainder when 7^{2023} is divided by 1000?
15. (70 points) Let $f(x)$ denote the number of trailing zeros in $x!$. For how many 3-digit integers x is $f(x+5) - f(x)$ an odd number? Note that $x! = x(x-1)(x-2)\cdots(2)(1)$.