Division B Individual Problems

1. What is the units digit of the product of any six consecutive integers?

2. A rhombus has perimeter 116 and diagonals that sum to 82. What is the area of the rhombus?

3. A *palindrome* is a number that remains the same when its digits are reversed. How many palindromes are smaller than 2023?

4. The product of four consecutive primes form the four-digit number $a00a$. What is the sum of these four prime numbers?

5. Andrew is competing in the CMC (Clown Math Competition), where a correct answer awards 0 points, a incorrect answer awards 6 points, and a blank answer awards 1.5 points. If there are 20 multiple choice questions with 12 possible answers each, and Andrew randomly guesses half the questions and leaves the rest blank, what is his expected score?

6. Find the value of $(21 + 4\sqrt{17})^{3/2} - (21 - 4\sqrt{17})^{3/2}$.

7. What is the largest prime factor of $3^{12} - 2^{12}$?

8. Let x be a 3-digit number in base b, and let y be a 2-digit number in the same base b. Given that $x - y = 1032_{10}$, and $x + y = 1150_{10}$, find the sum of all possible values of b.

9. A fly is tethered to a vertex of a solid cube of side length 2023 by a freely moving rope of length 6. Let S be the set of all points in space that the fly can reach. Find the absolute difference between the volume and surface area of S.

10. Find the largest power of 2 that divides

$$\sum_{n=0}^{2023} n(n + 1)(n + 2)(n + 3)(n + 4).$$