Division B SET 1 - 4 points per problem

- 1. At Gunn High School, 50% of students play video games, 40% of students play the piano, and 20% of the students who play the piano also play video games. What percent of non-piano players play video games?
- 2. What is the perimeter of the smallest non-degenerate scalene triangle with integer side lengths?
- 3. The Jokarul Factory loves to make counterfeit coins. Initially, there are 120000 real coins in the world. How many counterfeit coins should the factory manufacture if they want 25% of the world's coins to be counterfeit?

Division B SET 2 - 5 points per problem

- 4. The average of five consecutive even integers in increasing order is 90. What is the average of the first three numbers of those integers?
- 5. A bag contains 2 red balls and 3 blue balls. What is the probability that two consecutive draws (without replacement) from this bag will result in two balls of the same color? Express your answer as a common fraction.
- 6. A rectangle has area 52 and perimeter 34. Find the positive difference of the side lengths.

Division B SET 3 - 6 points per problem

- 7. What is $i + 2i^2 + 3i^3 + 4i^4 + \dots + 64i^{64}$?
- 8. A *palindrome* is a number that remains the same when its digits are reversed. What is the smallest four-digit palindrome that is also divisible by a three-digit palindrome?
- 9. How many lines of symmetry does a regular dodecagon (12 sides) have?

Division B SET 4 - 7 points per problem

- 10. Four integers a,b,c, and d are randomly chosen with replacement from between 0 and 2023, inclusive. What is the probability that (a d)(b c) is even?
- 11. Ayush the astronaut is still on a mission to find "Who asked?". Unfortunately, Ayush has just discovered that 4 small asteroids have punctured the spaceship. Fortunately, Ayush is in possession of an Advanced Self-Sustaining Bio-Regenerative Oxygenated Atmosphere Production Unit[™], a device that can produce an arbitrary amount of oxygen. Given that the 4 holes are emptying air at a rate of 5 minutes per liter, 6 minutes per liter, 10 minutes per liter, and 30 minutes per liter, respectively: What is the minimum speed, in minutes per liter, that Ayush should set the Atmosphere Production Unit[™] such that he does not run out of air?
- 12. Sophie is taking a 6-question multiple choice exam, and since he is randomly guessing, for each question he has an equal chance of getting 1, 3, or 5 points for his answer. How many times more likely is it for him to get a total score of 26 compared to a total score of 28?

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Division B SET 5 - 8 points per problem

- 13. Allan has 100 coins, some of which have a weighted with a probability 0.6 landing heads while the rest are fair. If the expected number of heads after tossing all 100 coins is 59, how many fair coins does Allan have?
- 14. 2023²⁰²³ is fully expanded and written down on a large whiteboard. The sum of digits is calculated. Then, the sum of the digits of the result is calculated, and so forth until we are left with a single digit. What is that digit?
- 15. Find the largest n such that the last three digits of n! are 800.

Division B SET 6 - 10 points per problem

- 16. Compute $3\sqrt{9\sqrt{27\cdots}}$, where each subsequent square root contains the next power of 3.
- 17. On a 6 by 6 grid of points, every point is to be colored as one of the three colors: yellow, green, or blue. What is the expected number of rectangles with all four vertices having the same color? (We only count rectangles having one pair of sides parallel to the x-axis and the other pair parallel to the y-axis).
- 18. How many four-digit even positive integers have four different digits, with 5 being the largest digit?

Division B SET 7 - 12 points per problem

- 19. Roger has a regular die with 6 sides. He rolls the die until he rolls a 1, a 5, or two consecutive rolls with even numbers. Compute the expected number of rolls Roger will need to achieve his goal.
- 20. An equilateral triangle of side length $\frac{2}{3}$ is dropped onto an infinite grid of unit squares. Given that the triangle lands such that one of its sides is parallel to one of the axes of the grid, what is the probability that the triangle doesn't intersect any grid lines?

21. Jinwoo and Steve are playing a coin game. The game works as follows:

- The players take turns flipping coins.
- The winner of the game is the player that flips the first head.
- The winner of the previous game flips first.

Assuming Jinwoo flips first in the first game, What is the probability that Jinwoo wins the 5th game?

Division B SET 8: ESTIMATION - 15 points max per problem

- 22. The chance that Alan touches grass today if he did not touch grass the previous day is 50%. However, if he did touch grass yesterday, then the chance he touches grass drops to 25%. Given that Alan touched grass today, what is the expected number of times he will touch grass over the following 23 days?
- 23. Every edge of a regular tetrahedron is colored one of 10 colors. If two colorings are the same if one can be rotated into the other, then how many colorings are there?
- 24. How many ways are there to place 23 queens on a 23×23 chessboard such that no two queens attach each other? Note: a queen attacks any number of squares vertically, horizontally, or diagonally.