## GMC Team B

1. (20 points) Let $i = \sqrt{-1}$. Simplify $i^2 + i^8$.

2. (20 points) Evaluate 1111111^2.

3. (20 points) If $a@b$ is $(a-b)(a+b)$ for positive integers a, b, what is $5@2$?

4. (25 points) In a triangle ABC with integer side lengths and positive area, $AB = 15$ and $BC = 7$. Find the number of possible values for the length of side AC.

5. (35 points) The units digit of a two-digit number is 3. The tens digit of that two-digit number is x and when the two-digit number is reversed, the differences of the two-digit numbers are 27. What is x?

6. (35 points) Samuel, Jerry, Ethan, Josh, and David have all just finished a math test. Samuel, Jerry, Ethan, and Josh scored a 91, 94, 92, and 93, respectively. However, David decided that the group's average score was too high, and he did badly on the test so that the average score would be below 90. What was the highest possible score David got?

7. (40 points) If 15 consecutive even numbers sum to 300, what is the largest number?

8. (40 points) A magic product square is an $N \times N$ grid of squares with each square containing a positive integer such that the product of the numbers in every row, column and main diagonal of the square is the same. A certain 3×3 magic product square has this common product equal to 5832. What integer is in the middle square of the grid?

9. (40 points) Find the units digit of 17^{123}.

10. (45 points) What is the sum of the *real* roots of the cubic polynomial $x^3 - x^2 - x - 2$?

11. (45 points) In a rectangular prism $ABCDEFGH$ shown below, the length of the space diagonal AH is 97 and the length of diagonal AC is 65. What is the length of CH?
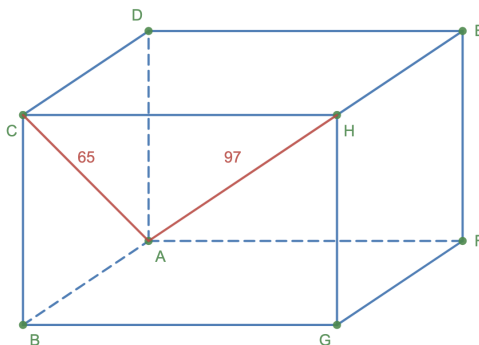
12. (50 points) Roger the rat wants to meet Saumya the snake for a lunchtime tea party. Roger is currently 6 blocks south and 4 blocks west of where Saumya currently is. However, Roger despises the giant cheesetrap 3 blocks north and 2 block east of where he is currently, and will not travel through that intersection. How many paths going only north and east can Roger follow to meet Saumya?
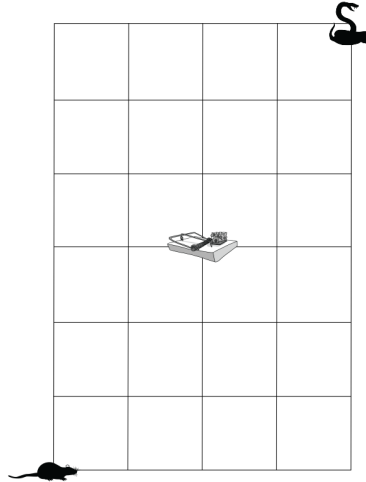
13. (55 points) What is the sum of the integer solutions of the equation $(x^2 + 13x + 21)^{(x^2-6x+8)} = 1$?

14. (60 points) What is $\sqrt[3]{3^3 + 4^3 + 5^3 + 6^3 + ... + 22^3}$?

15. (70 points) An equiangular hexagon has consecutive side lengths of 3, 4, 6, and 6. Then the area of this hexagon can be written as $\frac{a\sqrt{b}}{c}$, where a, b, c are integers and b is not divisible by the square of any prime. Find $a + b + c$.